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On the Path Length of Postural Sway

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Remarques sur la longueur du statokinésigramme

La longueur du statokinésigramme est la distance parcourue par le centre de pression des pieds d'un sujet debout. Bien que ce paramètre soit fréquemment utilisé en analyse du signal statokinésimétrique, il présente plusieurs inconvénients méconnus. Non seulement il ne fait que de répéter les informations déjà contenues dans les estimateurs des moments de la distribution statistique sous-jacente, mais, plus grave, les estimateurs usuels de ces moments sont probablement plus précis.

A stochastic process, like postural sway, is usually investigated through a study of the underlying moments of its statistical distribution. There are well-known estimators for the average values, which are the first-order moments, and the covariance functions or power spectra, which are the second-order moments. The properties of these estimators have been carefully studied; even their asymptotic distributions are known.

In the analysis of postural sway, some special statistics are used which are not part of the typical analysis of a stochastic process. One example is sway path (Hufschmidt, et al., 1980), which is the distance traversed by the center of foot pressure during a prescribed period of time. Although this statistic is often used (Okubo, Watanabe et al., 1979; Taguchi, et al., 1980; Watanabe, Okubo and Ishida, 1980; Koddleetal., 1982; Diener et al., 1984), it is, nevertheless, difficult to understand its theoretical justification. As I will show, this statistic only reduplicates information which is already contained in the estimators of the covariance function and, more importantly, the usual estimators of the covariance function are probably more efficient.

I will consider sway in only one direction, say, the anterior-posterior direction. Let Xi be the ith sample of the sway in that direction. If there are a total of N + 1 samples, then the sway path (SP) is defined as follows:

Agressologie, 1986,27,5: 431-432

$$SP = \sqrt{\frac{\pi}{2N}} \sum \sqrt{\left(X_{i+1} - X_i\right)^2}$$

Where the coefficient, $\sqrt{\frac{\pi}{2N}}$, was included in this definition to simplify subsequent results.

I will assume that postural sway is a stationary Gaussian process. I will set the average value of each Xi equal to zero for convenience. The auto covariance function, which I will denote as A_T is defined as follows:

AT = < xixi + T>

Where the brackets, in this and subsequent expressions, indicate an ensemble average. It is not difficult to show that the average sway path, <SP>, is given by the following expression:

<SP> = √Ao - A1

So the sway path is an unbiased estimator of the square root of the difference in the first two elements of the auto covariance function.

There are other statistics, which I can use to estimate the same quantity. For example, consider the following statistic, which I will call f.

$$f = \sum \left(X_{i+1} - X_i\right)^{\frac{2}{2N}}$$

It is easy to show that f consists of the difference in the standard estimators of Ao and A1 and that the average value of f is just equal to the <SP>^{2}. So I can use either statistics, SP or f, to obtain essentially the same information about postural sway, even though they do not share the same geometrical interpretation.

In order to compare the precision of these two statistics, I must determine their standard errors. For sway path, the expression for this error is quite complicated and cannot be written in terms of elementary functions without an additional assumption, but this assumption is quite reasonable.

Agressologie, 1986,27,5: 431-432

The statistical correlation, ς, between the path, $X_{i+1} - X_i$, and the path $X_{j+i} - X_j$ is related to the covariance function by the following expression:

$$\varsigma^2 = (2A_T - A_{T+1} - A_{T-1})/2\,(A_o - A_1)$$

Where T = i - j. In view of the last equation, the statistical correlation between path lengths from one increment to another will be very small if the auto covariance function is relatively smooth. If I neglect this correlation, the standard error for the sway path (SESP) is given by the following expression:

$$SESP = \langle Sp \rangle \sqrt{(\pi - 2)/2N}$$

The standard error for the statistic, f, which I will label as SEf, is :

$$SEf = \langle f \rangle \sqrt{2/N}$$

Since average value of f equals the square of the average value of SP, the two statistics are not on comparable scales. Hence the two standard errors cannot be compared directly. But I will compensate for this difference in the following way. Consider the average-value of f and a value, which is one standard error greater (or less) than this value and then take the square root of both of these values. The difference in these numbers can now be compared to the standard error of the sway path. This difference, which I will call SED, is given by the following approximate expression:

$$SED = \langle SP \rangle \sqrt{1/2N}$$

Notice that the standard error for the sway path (SESP) is larger than the comparable standard error (SED) for the statistic f. So, it is not sensible to use the statistic anterior-posterior sway path, since another more precise statistic is available.

This derivation points out important weaknesses in the statistic, sway path, which are not generally recognized. A theoretical basis has not been established for this quantity. Because of the square root in its definition, it is difficult to determine the analytic properties of this statistic. It certainly does not provide any new information about postural sway which is not already contained in the standard estimates of the moments of this stochastic process. And the more standard approaches are probably more precise.

Agressologie, 1986,27,5: 431-432

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SUMMARY

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Sway path is the distance traversed by the center of foot pressure for a standing subject. Although this statistic is frequently used in the analysis of postural sway, it has several unrecognized drawbacks. This statistic only reduplicates information, which is already contained in the estimators of the moments of the underlying statistical distribution, and, more importantly, the usual estimators of these moments are probably more precise.

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